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Engineering Notes

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H Flutter Analysis: A Direct Harmonic Interpolation Method

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Nomenclature

C_i = coefficient of spline interpolation
E = core function
G_c = coordinate on S_c
g = reduced damping
K_c = coordinate on S_c
k = reduced frequency
l_{ref} = reference length
N = number of support points
R_c = core size
r = distance
S_c = surface of core
V = speed, m/s
$\triangle$ = Laplace operator
δ = delta function

Subscript

m = support point number

I. Introduction

FLUTTER analysis is usually performed using two methods. The first is the k class [1] method, which predicts the correct flutter instability; however, the damping and frequency trends of the k method are known to be false and other anomalies might appear [2]. The second is the pk class [3] method, which, in addition to the k method, predicts the damping and frequency trends fairly correctly near zero damping.

These methods are usually based on generalized aerodynamic forces obtained for purely oscillatory motions. The prediction of the damping and frequency trends can be further improved by methods belonging to the pk class, such as the g method [4] and the p method [5]. The g method improves the damping and frequency trends of the pk method automatically near zero damping by taking into account the derivative of the generalized aerodynamic forces with respect to the damping at zero damping. The p method improves the damping and frequency trends by taking into account the effect of nonzero damping by means of generalized aerodynamic forces that are approximately valid for the damping-frequency area under consideration. However, methods (e.g., [6,7]) that generate the aforementioned forces hardly exist. In general, an analytical continuation of the generalized aerodynamic forces is applied with approximation errors as side effect due to the fitting procedures [8,9] associated with the generalized aerodynamic forces for purely oscillatory motion.

The more recent μ [10] flutter analysis method belongs to the p class and also relies on a fitting procedure to transform their aerodynamics to the state space. A novel flutter analysis method belonging to the pk class is introduced and described in this work. This so-called H method automatically extends the aerodynamic data obtained for purely oscillatory motions to damped and diverging oscillatory motions by means of a direct harmonic interpolation method, thereby improving the prediction of dampings and frequencies. The H method will be described and verified for a pitching flat plate. The results of a flutter analysis application will be presented for the well-known AGARD flutter test case.

II. Direct Harmonic Interpolation Model

This section describes the interpolation/continuation method with respect to the generalized aerodynamic forces. To obtain the generalized aerodynamic forces for nonzero dampings, the generalized aerodynamic forces, which are computed for zero damping, have to be warped to the nonzero damping space. Therefore, an interpolation is needed that provides implicitly the analytical continuation. Methods based on the class of spline techniques that are robust, automatic, and cardinal are used. For a theoretical background on the spline techniques, consult [11,12]. Hounjet and Meijer [11] introduce the volume spline and various core functions and discuss their behavior and implementation aspects extensively. Hounjet and Eussen [12] deal with recent developments.

Supposing the generalized aerodynamic forces GAF$(0, k_m)$ with respect to purely oscillating motions are calculated for N distinct frequencies k_m, we interpolate the data by:

$$\mathrm{GAF}\,(g,k) = C^0 + C^g g + C^k k + \sum_{m=1}^{N} C_m E(g,k;0,k_m) \tag{1}$$

where C are the coefficients that are determined by satisfying the aforementioned equation at the N support points m and additional closure relations:

$$\mathrm{Re} \sum_{m=1}^{N} C_m = 0 \tag{2}$$

$$\mathrm{Re}\, C^g = \mathrm{Im} C^k \tag{3}$$

$$\mathrm{Im}\ \mathrm{GAF}(g,0) = 0 \quad \forall\ g \tag{4}$$

$$\mathrm{Im} \sum_{m=1}^{N} C_m k_m = 0 \tag{5}$$

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$$\mathrm{Im}\, C^0 = 0 \tag{6}$$

$$\mathrm{Im}\, C^g = \mathrm{Re}\, C^k = 0 \tag{7}$$

The linear problem governed by Eqs. (1–7) is solved separately for the real and the imaginary parts. In particular, Eq. (4) is satisfied for the real and the imaginary parts by assuming a symmetric and antisymmetric C_m distribution with respect to the g plane, respectively.

In this work, it is required that the interpolation is harmonic, meaning that the kernel function E satisfies the Laplace equation in a two-dimensional space spanned by the reduced damping g and the reduced frequency k:

$$\triangle E = \delta g \delta k \tag{8}$$

Two types of kernels are considered:

Discrete source kernel: The Laplace kernel is consistently regularized according to an analogy with the determination of the autoinfluence of a Laplace field panel as developed in [13]:

$$E(g,k;0,k_m) = \begin{cases} \frac{1}{2\pi} \ln \sqrt{(k-k_m)^2+g^2} & \sqrt{(k-k_m)^2+g^2} \geq R_c; \\ \bar{E}(0,k_m;0,k_m)\left(1-\frac{\sqrt{(k-k_m)^2+g^2}}{R_c}\right) + E(G_c,K_c;0,k_m)\frac{\sqrt{(k-k_m)^2+g^2}}{R_c} & 0 < \sqrt{(k-k_m)^2+g^2} < R_c \end{cases} \tag{9}$$

with

$$\bar{E}(0,k_m;0,k_m) = \frac{1}{2\pi} \ln R_c - \frac{1}{\pi} \tag{10}$$

$R_c = \sqrt{(K_c - k_m)^2 + G_c^2}$ denotes the core size, which is taken as the minimum distance between the support points, and G_c and K_c are locations on the cylinder with size R_c. The singular kernel is regularized by redefining the value of $E(0, k_m; 0, k_m)$ in the form of a weighted sum of neighboring values and by linear interpolation E between $r = 0$ and $r = R_c$. The following property is used in redefining/regularizing the value of E at the origin:

$$\int_{S_c} \Delta E \mathrm{d}k\, \mathrm{d}g = 1 \tag{11}$$

where S_c denotes the cylinder with size R_c.

Continuous linear source kernel: As an alternative, a distributed core is applied that avoids the aforementioned regularization. The distributed core chosen here is a linear tentlike distribution of source singularities through successive frequency support points.

III. Verification

To verify the direct harmonic interpolation method, use is made of the d2d1 doublet lattice method [14], which operates for harmonic frequencies and positive g using the three-dimensional kernel integrated along the whole y axis. For negative g, the method does not converge.

Results obtained with this new approach are presented in Figs. 1 and 2 for a flat plate that performs a pitching motion about the quarter chord at a Mach number of 0.8. The wing normal force coefficient (CN) and moment about the quarter-chord coefficient (CM) have been calculated. The d2d1 method is first applied for the pitching

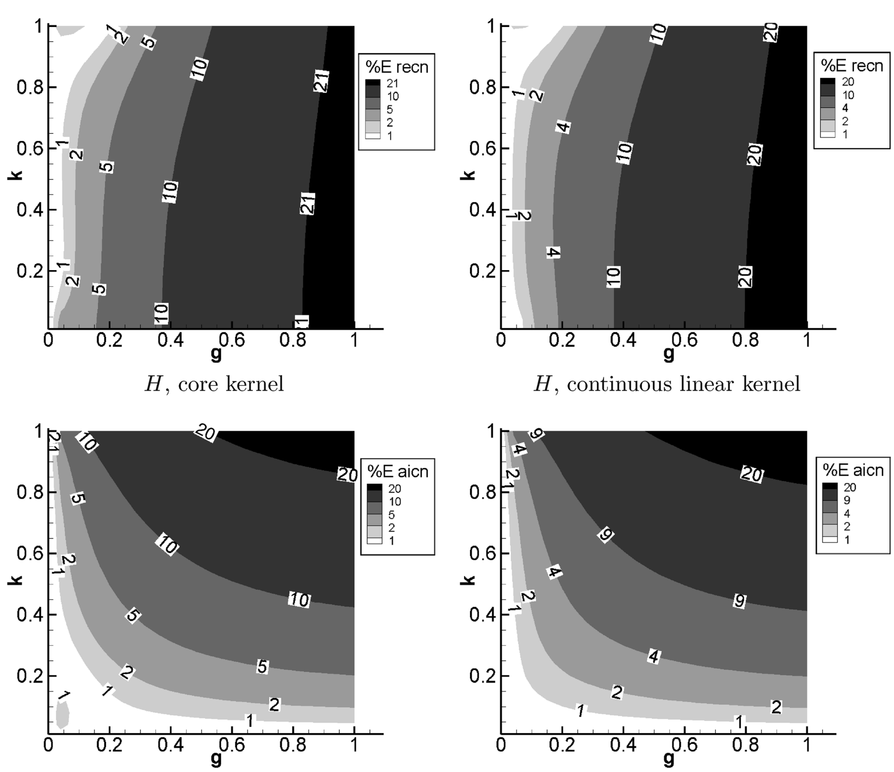

Fig. 1 Relative error in the real and imaginary parts of the normal force coefficient of a pitching flat plate at Mach = 0.8 for the discrete source and continuous linear source kernels.

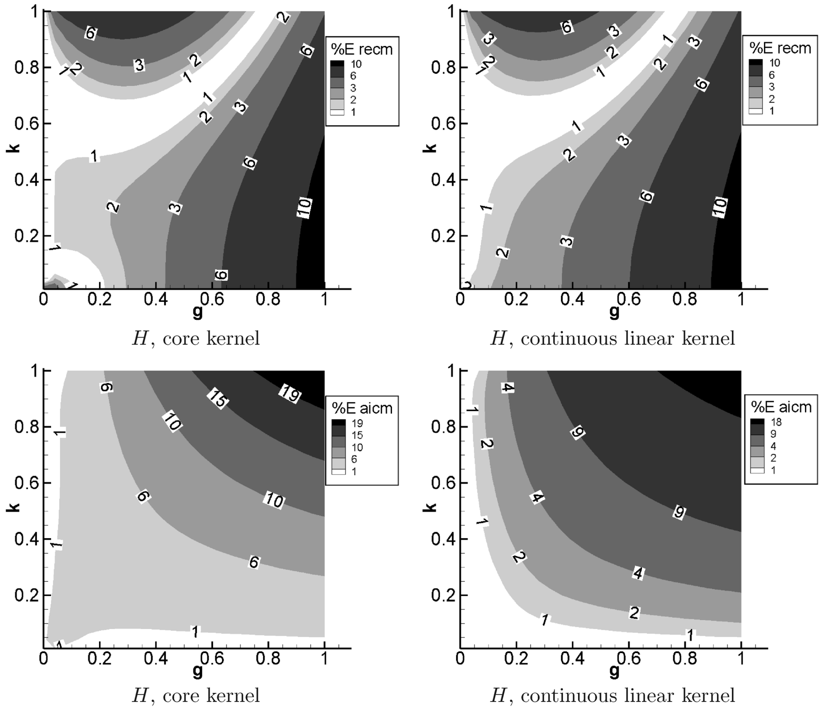

Fig. 2 Relative error in the real and imaginary parts of moment about the quarter-chord coefficient of a pitching flat plate at Mach = 0.8 for the discrete source and continuous linear source kernels.

flat plate in the ranges of $g = 0 \cdots 1$ and $k = 0 \cdots 1$ with a step size of 0.04. The selected range is typical for aeroelastic studies. Next, the data for $g = 0$ is used by the aforementioned direct harmonic interpolation method and warped to $g \neq 0$ with the discrete kernel and the continuous kernel, respectively.

Figures 1 and 2 show a comparison of the original data and the warped data using the aforementioned core and linear kernels in terms of the relative error in percentages. Figure 1 shows a contour plot of the relative error in the real part and the imaginary part of the lift coefficient, respectively. Figure 2 shows a contour plot of the relative error in the real part and the imaginary part of the moment about the quarter-chord coefficient, respectively. At reduced dampings approaching zero, the error is very small for both kernels. The simpler-to-apply discrete core kernel is almost level with the continuous core kernel. Fairly good agreement is obtained even for large values of g. A further reduction of the differences might also be obtained by using more points along the interval or increasing the borders and/or with better suited conditions (radiation) at the outer borders and/or adding support points at the real axis.

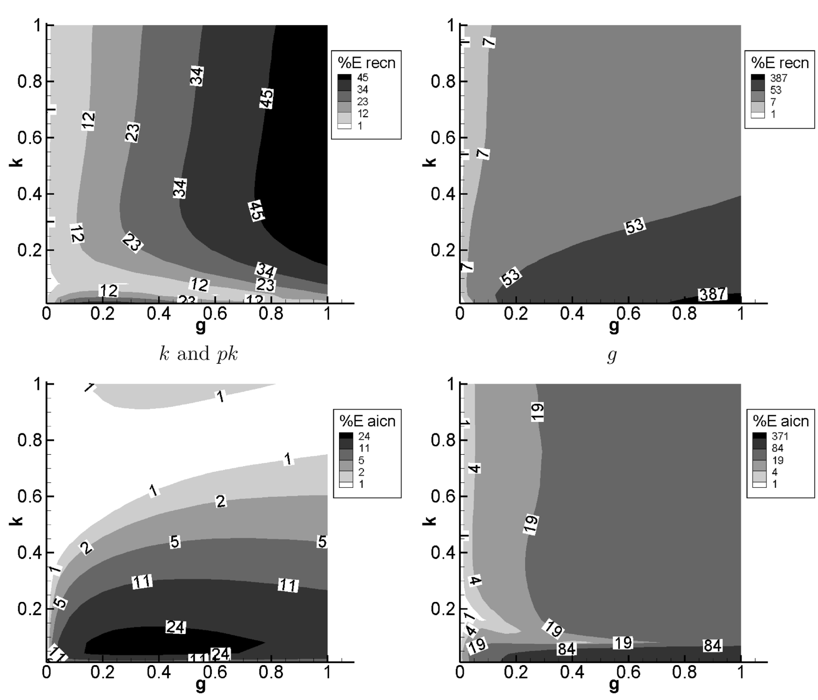

Fig. 3 Relative error in the real and imaginary parts of the normal force coefficient of a pitching flat plate at Mach = 0.8 for the *k* and *pk* methods and the *g* method.

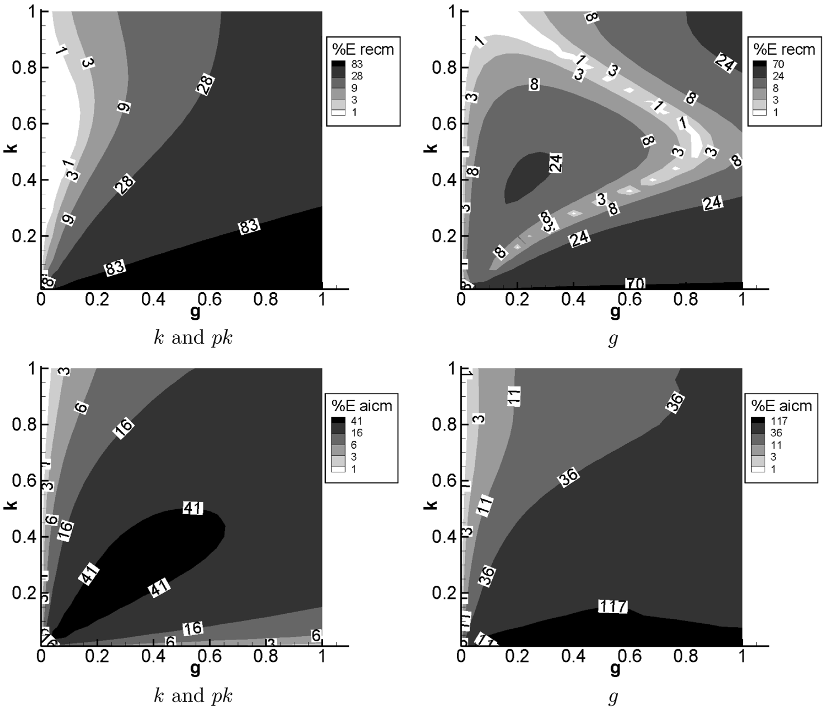

Fig. 4 Relative error in the real and imaginary parts of moment about the quarter-chord coefficient of a pitching flat plate at Mach = 0.8 for the *k* and *pk* methods and the *g* method.

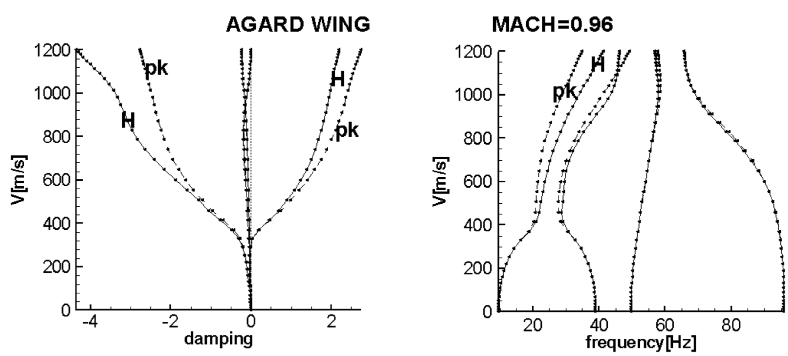

Fig. 5 Flutter diagram for AGARD wing 445.6 at a Mach number of 0.96 using the *pk* method (dashed) and the *H* method (solid).

Finally, the relative error associated with the k, pk, and g methods are presented in Figs. 3 and 4. The k and pk methods assume that the generalized forces are invariant with respect to g. The g method assumes $\mathrm{GAF}(g,k) = \mathrm{GAF}(0,k) + g\frac{\partial \mathrm{GAF}(0,k)}{\partial k}$. Both approaches show larger error levels compared with the error levels observed for the H method.

IV. Flutter Application

The results of the H method for the AGARD wing at a Mach number of 0.96 are compared in Fig. 5 with the results obtained with the pk method. The unsteady aerodynamic data for these analysis are computed using lifting surface theory. The pk method applies an iterative procedure without complete eigenvalue solutions based on the relation between the eigenvalues, the determinant, and the characteristic polynomial of the governing matrix. The only difference required for the H method is that the k interpolation of the GAFs is replaced by the g, k interpolation of Eq. (1). Both methods predict the same flutter instability mechanism. The dampings ($=g/k$) and frequencies ($=kV/2\pi l_{\mathrm{ref}}$) of both methods agree up to high levels of the dampings and velocity, thus affirming the well-known fact that results of the pk method are fairly correct beyond zero damping. The H method seems to lower the damping levels and predict a tighter connection between the flutter mode shapes after the flutter point has been passed.

V. Conclusions

A novel flutter analysis method, called the H method is introduced. The H method contains a simple procedure that automatically extends the aerodynamic forces data obtained for purely oscillatory motions to damped and diverging oscillatory motions by means of fitting free interpolation. The fitting free interpolation is described and verified with fairly good success for a two-dimensional flat plate. In addition, flutter analysis results for AGARD wing 445.6 demonstrate, in comparison with the pk method, a different damping trend capturing at high levels of dampings. This procedure may help the aeroelastician in making improved estimates of aerodynamic dampings to support flight flutter testing and probably offers potential for flight control system design/analysis.

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